CS646: DL Reasoning

NOTE: This document is also available at `http://www.cs.man.ac.uk/~horrocks/Teaching/cs646/Labs/dlreasoning/`.

1 OWL Abstract Syntax

As well as the rather verbose RDF/XML syntax, OWL also has a "human readable" syntax which is (rather oddly) called the *abstract syntax*. The abstract syntax corresponds very closely to the ontology structure presented by Protégé, i.e., Class and Property "definition" axioms, plus axioms to capture additional information such as disjointness and other background facts not represented in the "definitions". A complete specification of the abstract syntax can be found at `http://www.w3.org/TR/owl-semantics/syntax.html`; some examples of the abstract syntax and equivalent RDF/XML syntax can be found at `http://www.w3.org/TR/owl-semantics/examples.html#B.1`.

Consider the following small ontology written in OWL abstract syntax:

Class(Animal partial)
Class(Plant partial)
DisjointClasses(Animal Plant)
ObjectProperty(eats domain(Animal))
Class(Herbivore complete super(restriction(eats allValuesFrom(Plant))))
Class(Carnivore complete super(restriction(eats allValuesFrom(Animal))))
Class(CarnivorousPlant complete super(Plant) super(Carnivore))

The first axiom, i.e., Class(Animal partial), can be translated into an equivalent DL axiom and into equivalent FOL as follows:

DL: $\text{Animal} \sqsubseteq \top$
FOL: $\forall x.\text{Animal}(x) \rightarrow \text{True}$

Write down DL and FOL equivalents of each of the other axioms in the ontology.

2 DL Reasoning

1. Enter the above ontology into Protégé, and use the reasoner to compute the class hierarchy.
2. Explain why Plant is a sub-class of both Herbivore and Carnivore.
3. This does not seem to be correct. Explain how you would improve the ontology in order to fix this problem.
4. Did your improvement reveal any other problem with the ontology? If so, how would you repair the problem?

3 DL Semantics

Consider the following small Knowledge Base (ontology) $\mathcal{K}$, where $\mathcal{K}$ consists of the following set of axioms:

$$
\begin{array}{rcl}
\text{Person} & \sqsubseteq & \top \\
\text{Man} & \sqsubseteq & \text{Person} \\
\text{Woman} & \equiv & \text{Person} \sqcap \neg\text{Man} \\
\text{Parent} & \equiv & \text{Person} \sqcap \exists\text{hasChild.Person} \\
\text{Father} & \equiv & \text{Man} \sqcap \text{Parent} \\
\text{Mother} & \equiv & \text{Woman} \sqcap \text{Parent} \\
\text{TwoChildFather} & \equiv & \text{Father} \sqcap \geqslant 2\text{hasChild} \sqcap \leqslant 2\text{hasChild} \\
\text{HappyFather} & \equiv & \text{Father} \sqcap \exists\text{hasChild.Man} \sqcap \exists\text{hasChild.Woman} \\
\text{TiredParent} & \equiv & \text{Parent} \sqcap \geqslant 5\text{hasChild} \\
\text{hasChild} & \sqsubseteq & \text{hasDescendant} \\
\text{hasDescendant}^{+} & \sqsubseteq & \text{hasDescendant} \\
\text{hasAncestor} & \sqsubseteq & \text{hasDescendant}^{-} \\
\text{John} & : & \text{Man} \\
\text{Mary} & : & \text{Woman} \\
\langle \text{John}, \text{Mary} \rangle & : & \text{hasChild}
\end{array}
$$

and the interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ of $\mathcal{K}$, where

$$
\begin{array}{rcl}
\Delta^{\mathcal{I}} & = & \{a, b, c, d, e, f, g, h\} \\
\text{John}^{\mathcal{I}} & = & a \\
\text{Mary}^{\mathcal{I}} & = & b \\
\text{Person}^{\mathcal{I}} & = & \{a, b, d, f, g, h\} \\
\text{Man}^{\mathcal{I}} & = & \{a, g, h\} \\
\text{hasChild}^{\mathcal{I}} & = & \{(a, b), (a, h), (b, d), (g, a), (g, f)\}.
\end{array}
$$

1. Extend $\mathcal{I}$ to give extensions to the other concepts (classes) in $\mathcal{K}$ and roles (properties) in the Knowledge Base (ontology), i.e., complete the following:

$$
\begin{array}{rcl}
\text{Woman}^{\mathcal{I}} & = & \\
\text{Parent}^{\mathcal{I}} & = & \\
\text{Father}^{\mathcal{I}} & = & \\
\text{Mother}^{\mathcal{I}} & = & \\
\text{TwoChildFather}^{\mathcal{I}} & = & \\
\text{HappyFather}^{\mathcal{I}} & = & \\
\text{TiredParent}^{\mathcal{I}} & = & \\
\text{hasDescendant}^{\mathcal{I}} & = & \\
\text{hasAncestor}^{\mathcal{I}} & = &
\end{array}
$$

2. Is the extension of any of the concepts empty? If so, does this mean that these concepts are unsatisfiable? Explain your answer.

3. Is the extension of HappyFather empty? If not, does this mean that HappyFather is satisfiable w.r.t. $\mathcal{K}$? Explain your answer.

4. Is the extension of TwoChildFather equal to the extension of HappyFather? If so, does this meant that the two concepts are equivalent w.r.t. $\mathcal{K}$ (i.e., that $\mathcal{K} \models \text{TwoChildFather} \equiv \text{HappyFather}$)? Explain your answer.

5. Is the extension of John in the extension of HappyFather? Does this meant that John is and instance of HappyFather w.r.t. $\mathcal{K}$ (i.e., $\mathcal{K} \models \text{John} : \text{HappyFather}$)?

4 Ontology Examples

Load the People and Pets ontology (`http://www.cs.man.ac.uk/~horrocks/Teaching/cs646/Labs/PeopleAndPets.owl`) into Protégé. Work through the examples in the *Reasoning with OWL* lecture notes (`http://www.cs.man.ac.uk/~horrocks/Teaching/cs646/Slides/why.ppt`), checking the various inferences using Protégé and making sure you understand why the inferences are being drawn.