



Logic for Computer Science. Knowledge Representation and Reasoning.

Lecture Notes
for
Computer Science Students
Faculty EAIIB-IEiT AGH



Antoni Ligęza

Other support material:

<http://home.agh.edu.pl/~ligeza>

<https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start>

Multi-Valued Logics

In classical **Propositional Calculus** we have just 2 truth-values; something can be **True** or **False** (technically: 1 and 0):

$$I: P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

In **Multi-Valued Logics** there can be 3 (or more) values.

The first 3-valued logic was introduced by **Jan Łukasiewicz** in 1920.

$$I: P \longrightarrow \{0, \frac{1}{2}, 1\},$$

The meaning of $\frac{1}{2}$ is **unknown**; maybe becoming true or false in future.

The truth-tables are based on the following practical formulas:

- $I(\neg p) = 1 - I(p)$,
- $(p \wedge q) = \min(I(p), I(q))$,
- $I(p \vee q) = \max(I(p), I(q))$,
- $I(p \rightarrow q) = \min(1, 1 + I(q) - I(p))$.

In Relational Databases/SQL:

- **NULL** – unknown but existing value (date of birth),
- **NULL** – unknown, maybe not existing value (no. of telephone)
- **NULL** – value of an attribute not applicable to an object

AND	<i>TRUE</i>	<i>FALSE</i>	<i>NULL</i>	OR	<i>TRUE</i>	<i>FALSE</i>	<i>NULL</i>								
<i>TRUE</i>	<i>TRUE</i>	<i>FALSE</i>	<i>NULL</i>	<i>TRUE</i>	<i>TRUE</i>	<i>TRUE</i>	<i>TRUE</i>								
<i>FALSE</i>	<i>FALSE</i>	<i>FALSE</i>	<i>FALSE</i>	<i>FALSE</i>	<i>TRUE</i>	<i>FALSE</i>	<i>NULL</i>								
<i>NULL</i>	<i>NULL</i>	<i>FALSE</i>	<i>NULL</i>	<i>NULL</i>	<i>TRUE</i>	<i>NULL</i>	<i>NULL</i>								
<table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 5px;">NOT</th> <th style="border-bottom: 1px solid black; padding: 5px;"><i>TRUE</i></th> <th style="border-bottom: 1px solid black; padding: 5px;"><i>FALSE</i></th> <th style="border-bottom: 1px solid black; padding: 5px;"><i>NULL</i></th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; padding: 5px;"><i>TRUE</i></td> <td style="padding: 5px;"><i>FALSE</i></td> <td style="padding: 5px;"><i>TRUE</i></td> <td style="padding: 5px;"><i>NULL</i></td> </tr> </tbody> </table>								NOT	<i>TRUE</i>	<i>FALSE</i>	<i>NULL</i>	<i>TRUE</i>	<i>FALSE</i>	<i>TRUE</i>	<i>NULL</i>
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Fuzzy Logic

Let U be a classical set (a universe). Any subset X of U can be defined by the so-called **characteristic function** – a **predicate** – m :

$$m: U \rightarrow \{0, 1\}$$

so that $m(x) = 1$ iff $x \in X$.

A **Fuzzy Set** A defined in U is a pair $A = (U, \mu_A)$, where:

$$\mu_A: U \rightarrow [0, 1]$$

In classical **Propositional Calculus** we have just 2 truth-values; something can be **True** or **False** (technically: 1 or 0):

$$I: P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

In **Fuzzy Logic** there can infinitely many truth values belonging to the interval $[0, 1]$.

The notion of **Fuzzy Sets** and **Fuzzy Logic** was introduced by **Lotfi Zadeh** in 1965.

$$I: P \longrightarrow [0, 1],$$

The meaning of $I(p) = \alpha$ for $0 < \alpha < 1$ is that p is **partially true**.

The truth-tables are based on the following practical formulas:

- $I(\neg p) = 1 - I(p)$,
- $(p \wedge q) = \min(I(p), I(q))$,
- $I(p \vee q) = \max(I(p), I(q))$,
- $I(p \rightarrow q) = \min(1, 1 + I(q) - I(p))$.

Temporal Logics

In classical **Propositional Calculus** we have just 2 truth-values; something can be **True** or **False** (technically: 1 or 0):

$$I: P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

The **logical value** of any proposition $p \in P$ remains true or false over all the time of concern. In other words., the truth values of formulas does not change over time.

In **dynamic systems** the state – and so its description – does change over time.

In the simplest Propositional Temporal Logic there are two temporal operators introduced:

- \square – with the meaning **always**; all the time, and
- \diamond – with the meaning **eventually**; somewhere in the future.

So the intended meaning is:

- $\square p$ – p holds all over the time (defining **safety**),
- $\diamond p$ – p will eventually happen (defining **liveness**).

Possible temporal models:

- continuous vs. discrete time
- intervals vs. instants (time points),
- linear vs. branching time,
- symbolic sequential models vs. real time models.